

# An artificial neural network base prediction model and sensitivity analysis for marshall mix design

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Digital Object Identifier (DOI): dx.doi.org/10.14311/EE.2016.224

## ABSTRACT

*This study presents an artificial neural network (ANN) model to predict the Hot Mix Asphalt (HMA) volumetrics of mixtures prepared by following Marshall mix design procedure. The input data set of the model is determined to be the aggregate gradation, bulk specific gravity of aggregates, and binder content of the mixture based on the available data. The proposed ANN model utilizes one-layer Levenberg-Marquardt backpropagation to predict the theoretical maximum specific gravity of the loose mixture (Gmm) and the bulk specific gravity of the compacted mix (Gmb). The ANN was trained using data obtained from numerous roads with a total of 835 different mix designs. The estimated HMA volumetrics, Gmb and Gmm, are used to calculate key design criteria such as percent of air voids (Va), Voids in the Mineral Aggregate (VMA), and Voids Filled with Asphalt (VFA). The results revealed that the ANN is able to predict volumetrics within a promising accuracy. The proposed ANN model was able to predict the Va within  $\pm 1.0\%$  range 90% of the time and within  $\pm 0.5\%$  range 55% of the time. The reasonable predictions of the model leads to significant time, cost and labor savings with respect to traditional Marshall Mix Design by limiting the number of trials to reach to the optimum mix design. With the developed ANN model, Marshall mix design can take 1.5 to 3 days with little validation effort in the laboratory. In addition, the model could be used as a practical Quality Control tool for roadway agencies to verify the mix designs.*

**Keywords:** Compaction, Mixture design, Voids

## 1. INTRODUCTION

The Marshall Mix Design procedure is one of the oldest and still common method for designing hot mix asphalt (HMA) pavements. The mechanical properties of asphalt concrete are evaluated through experiments and trial and errors on HMA specimens. The mix design procedure is empirical and depends on the lengthy laboratory experiments (ASTM D6926). The time spent on mix design significantly depends on the experience of the designer and facilities available in the laboratories. Therefore, the time spent on mix designs may affect the construction schedules, especially during the peak construction seasons. The proposed mix design procedure may lead significant savings. The main objective of this study is to come up with a computational tool that is based on an ANN to minimize the experimental work carried out during the asphalt mixture design. The proposed ANN model is used to predict the volumetric properties of a mix, such as the theoretical maximum specific gravity of the loose mixture ( $G_{mm}$ ) and the bulk specific gravity of the compacted mix ( $G_{mb}$ ). The inputs to the ANN model are (i) gradation of the mix, (No.1½"- No. 200) (ii) bulk specific gravity of aggregates ( $G_{sb}$ ), (iii) binder content of the mix ( $P_b$ ).

ANNs have been widely utilized to predict complex variables in diverse subjects in civil engineering applications. It is also widely used in the pavement area to efficiently predict data that are difficult to obtain without lengthy experiments or advance models. The deflection data gathered from falling-weight deflectometers were used to estimate the pavement moduli by training the ANNs in many research studies [1], [2],[3]. In another study, falling weight deflectometer data was used to determine the pavement structural condition in pavement management systems [4]. Similarly, pavement shear moduli were predicted by ANNs trained using the deviatoric and confining triaxial stresses measured from triaxial test, sample deformation and aggregate properties [5]. Resilient moduli data was trained by ANNs to predict the dynamic modulus master curve of asphalt mixtures [6]. Another ANN based model was developed to predict the dynamic modulus of HMA inputting aggregate shape parameters, frequency, asphalt viscosity and air voids of compacted samples [7]. An ANN based model was developed and trained to predict improved  $|E^*|$  predictions to better estimate the distresses for flexible pavements [8]. In various studies, ANN based models were also used to predict the fatigue life of pavements [9],[10]. Permeability of asphalt mixtures were estimated by an ANN model inputting the mixture properties such as air voids, aggregate distribution, degree of saturation and effective asphalt-to-dust ratio [11]. In some parts of the Mechanistic Empirical Design Guide (MEPDG), ANN models were also utilized such as crack growth algorithm in the software [12]. ANNs were also utilized to determine the severity and type of the cracks [13],[14],[15]. In another study, ANN model was used to segment the coarse aggregates from air voids and mastic in poor contrast X-Ray CT images [16]. ANN models were also developed to predict the Superpave asphalt mixture volumetrics at the initial, design and maximum design gyrations levels [17],[18].

This paper presents an ANN model developed to estimate the Marshall asphalt mixture volumetrics i.e., air void ( $V_a$ ), Voids in the Mineral Aggregate (VMA), and Voids Filled with Asphalt (VFA). Marshall mixture design is a laborious process that can take approximately a week when all the steps are carried out neatly. This model allows significant time and material savings of spent during a traditional Marshall mix design. In addition, another advantage of the model is that it may be used as Quality Control (QC) tool for Agency or Engineer to rapidly validate the job mix formula (JMF).

## 2. ARTIFICIAL NEURAL NETWORK (ANN) MODEL

A neural network is defined as a parallel-distributed processor with high-computation power made of various processing units, called neurons [19]. Artificial neural networks and biological (brain) networks have significant similarities, whereas biological neural networks are more complex and are composed of approximately  $10^{11}$  neurons [20]. Human brain works as a three-stage system. It receives information, understands it and makes appropriate decisions in two ways either feed-back or feed-forward. Similarly, a typical ANN structure is basically composed of three stages: input, one or more hidden and output layers. Initially, the input layer is for receiving the information. Secondly, the hidden layers are to perceive and cluster the information. Finally, the output layer is to give the decision/outputs. The determination of the number of hidden layers is crucial for the efficiency of the model and significantly depends on the diversity of the input data. With respect to complexity of the model, maximum two hidden layers are commonly recommended [19]. In the literature, there are many rules of thumbs to limit the number of neurons [21]. However, both the number of hidden layers and neurons depend on the complexity and structure of the data. If inadequate number of neurons is used in the ANN model, it delays the learning process of the model. On the contrary, the use of excessive number in the model forces the model to memorize rather than learning and understanding the data. Therefore, researchers suggested keeping a separate set of data to re-validate the ANN models [17],[18].

### 2.1. Marshall mix design input database

To develop the virtual mix design model, a large database of input–output pairs was needed. 835 Marshall mix design data from the HMA pavements constructed in Turkey were obtained from General Directorate of Highways. Then, the outliers caused by typos and incomplete/missing data were eliminated. In addition to statistical tools that were used to exclude the outliers, manual checks were done to ensure the accuracy of the data. As given in Table 1, the inputs of the developed model in this study are very limited and composed of (i) gradation of mix, (ii) bulk specific gravity of

aggregates ( $G_{sb}$ ), and (iii) binder content of the mix. The aggregate gradations in the data set vary from fine to coarse. Thus, the range of binder content in the mixtures also varies widely, providing a wide set of the data for training and validation of the ANN. The binder penetration grades for the mixtures are all PEN 50-70 in this study. The numbers of blows of Marshall mixes are either 50 or 75. However, the number of blow data is not available for all mixtures. Therefore, it is not included as input parameter to the model.

Before the training of the developed ANN model, the structured database was divided into two as: (i) training dataset and (ii) testing dataset. The training set was used to train the model, whereas the testing set used for the simulation/validation of the model. In addition, during the training process, the trained dataset is also divided into three subsets: (i) Train (70% of the dataset), (ii) Validation (15% of the dataset), and (iii) Test (15% of the dataset).

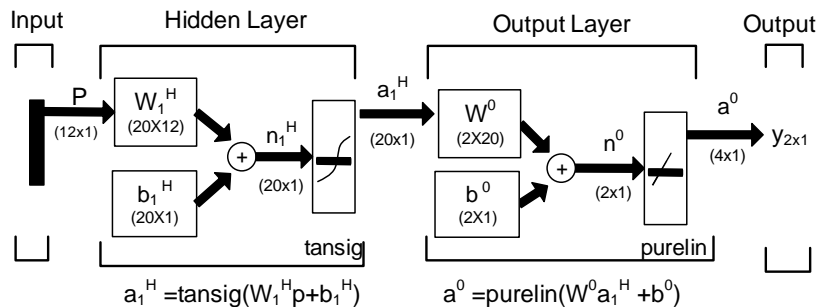
Of the 835 mix designs, 751 mix designs were selected randomly for the training process of the ANN model. The rest of the data (84 mix designs) were kept separate for the testing/independent validation of the developed ANN model. In other words, the validation of the model was performed twice, initially during the training stage, and secondly using 84 mix designs set aside after the training is completed. Ultimately, the model was trained to estimate theoretical maximum specific gravity of the loose mixture ( $G_{mm}$ ) and the bulk specific gravity of the compacted mix ( $G_{mb}$ ). Thus, two outputs can be used to calculate three crucial variables:  $V_a$ , VMA and VFA for Marshall Mix Design

**Table 1. Input Parameters used in the ANN model and range of values.**

Agg. Grad. (Sieve size, mm)	Percent Passing					Min	Max	Avg.	St. Dev.
	Min	Max	Avg.	St. Dev.					
37.5	100	100	100	0	$G_{sb}$	2.396	2.955	2.676	0.070
25.4	80.2	100	96.04	5.48	$P_b$	2.5	8	4.52	0.819
19.1	71.7	100	88.41	8.18					
12.7	55.1	100	72.66	10.82					
9.52	49.7	94	62.76	9.30					
4.76	30.1	63	43.03	3.34					
2	20.6	46.6	26.76	2.77					
0.42	7.5	19	11.66	1.54					
0.177	5	14.9	7.96	1.26					
0.075	3.2	12.2	5.19	0.96					

## 2.2. ANN Model structure

The developed model in this study utilizes one-layer Levenberg-Marquardt backpropagation ANN. As shown in Figure 1, the input layer composes of 12 inputs/neurons i.e. aggregate gradation,  $G_{sb}$  and  $P_b$ . The output layer consists of 2 outputs/neurons i.e.  $G_{mm}$  and  $G_{mb}$ . After many trial and errors, it is determined that one hidden layer with 20 neurons is the optimum solution for the developed model. For the ease of the illustration of the ANN structure, the abbreviated notations suggested in MATLAB User's Guide were used in Figure 1 [22]. 12x1 input vector ( $p$ ) is given to the model for a 2x1 output vector ( $y$ ).  $W^H$  and  $W^O$  are defined as weight factors in matrix form for hidden and output layers, respectively. Likewise,  $b^H$  and  $b^O$  are the bias factors in vector form, respectively.



**Figure 1: Structure of the ANN model.**

(i) The output of the hidden layer ( $a_1^H$ ) is calculated using Equation 1 and 2.

$$n_1^H = W_1^H * p + b_1^H \quad (1)$$

$$a_1^H = \text{tansig}(n_1^H) \quad (2)$$

where  $n_1^H$  is the net input vector and  $a_1^H$  is the output of the hidden layer. The "tansig" is the transfer function that is

computed using Equation 3.

$$\text{tansig}(n_1^H) = \frac{2}{1 + \exp(-2 * n_1^H)} - 1 \quad (3)$$

(ii) Similarly, the output layer is computed using Equation 4 and 5.

$$n^0 = W^0 * a_1^H + b^0 \quad (4)$$

$$y = \text{purelin}(n^0) \quad (5)$$

where  $n^0$  is the net output vector. The “purelin” is the linear transfer function. The output of the entire network ( $y$ ) is calculated by repeating the Steps (i) and (ii) for all input values (training dataset). The weight and bias values in the hidden and output layers are adjusted until the overall computed mean square error is less than  $10^{-4}$  or the number of epochs reaches to 5000. The mean square error (mse) between the computed  $y$  and the target  $y^t$  is computed for all the training dataset using Equation 6.

$$MSE = \frac{\sum_{i=1}^N (y_i - y_i^t)^2}{N} \quad (6)$$

where  $N$  is the number of data in the validation dataset inputted to the network. The developed ANN model is accomplished using the Levenberg-Marquardt backpropagation method (trainlm) provided in MATLAB neural network toolbox.

### 3. PERFORMANCE OF ANN MODEL

Performance of the ANN model was evaluated in two ways: (i) the predicted vs. measured values of  $G_{mb}$  and  $G_{mm}$  were plotted and coefficient of determination ( $R^2$ ) with respect to line of equality was computed, (ii) a basic parameter “Success Rate (SR)” is defined, in order to represent the percentage of the ANN predictions that are within a defined target error range. This parameter is used in case the  $R^2$  may be misleading due to clustering around the line of equality. As shown in Figure 2, predicted and measured data lie around the line of equality, and approximately 98% of the predicted data lay in the  $\pm$  prediction interval (PI) range. Considering the variability of laboratory-to-laboratory and technician-to-technician in the input and target dataset, the prediction performance of the ANN model appears reasonable since the data scattering is limited with  $R^2=0.94$  for  $G_{mm}$  and  $R^2=0.92$  for  $G_{mb}$ . As shown in Figure 3, SRs of the models are also promising for  $G_{mm}$  and  $G_{mb}$ . The SRs of predicted  $G_{mm}$  data are 67%, 96%, and 98% for error ranges of  $G_{mm} \pm 0.01$ ,  $G_{mm} \pm 0.03$ , and  $G_{mm} \pm 0.05$ , respectively. Similarly, the SRs of predicted  $G_{mb}$  data are 53%, 94%, and 99% for error ranges of  $G_{mb} \pm 0.01$ ,  $G_{mb} \pm 0.03$ , and  $G_{mb} \pm 0.05$ , respectively.

Table 3 shows the SRs for the percent of air voids ( $V_a$ ) for the errors of  $V_a \pm 0.5\%$  and  $V_a \pm 1.0\%$ . For air void predictions, there is 54.19% and 86.02% probability that the predicted ( $V_a$ ) will be within  $\pm 0.5\%$  and  $\pm 1.0\%$ , respectively. Hence, ANN model predictions are reasonable.

As specified before, 84 mix designs were separated for independent validation of the ANN model and were not included in the training dataset. The trained ANN model was used to estimate the  $G_{mm}$  and  $G_{mb}$  of these 84 independent mix designs/data. As shown in Figure 4, predicted versus measured values of specific gravities, where  $R^2$  values are 95% and 91 % for  $G_{mm}$  and  $G_{mb}$ , respectively. Similarly, the SR of the ANN model for the test data is also promising for specific gravity predictions as given in Figure 3. SRs of predicted  $G_{mm}$  data are 67%, 93%, and 100% for error ranges of  $G_{mm} \pm 0.01$ ,  $G_{mm} \pm 0.03$ , and  $G_{mm} \pm 0.05$ , respectively. Similarly, the SRs of predicted  $G_{mb}$  data are 46%, 94%, and 98% for error ranges of  $G_{mb} \pm 0.01$ ,  $G_{mb} \pm 0.03$ , and  $G_{mb} \pm 0.05$ , respectively. For air voids, there is 90% probability that the predicted  $V_a$  will be within  $\pm 1.0$ , and there is 56% probability that the predicted  $V_a$  will be within  $\pm 0.5\%$ , as given Table 2.

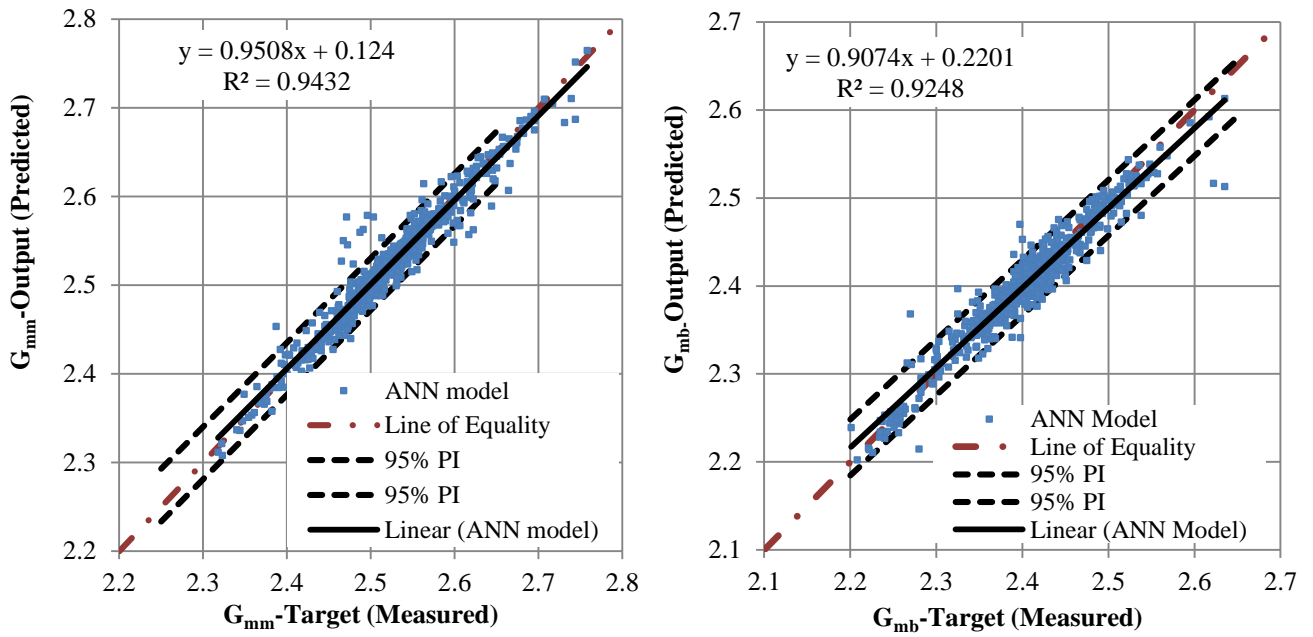


Figure 2: Predicted versus measured values of a)  $G_{mb}$ , b)  $G_{mm}$  for the training data

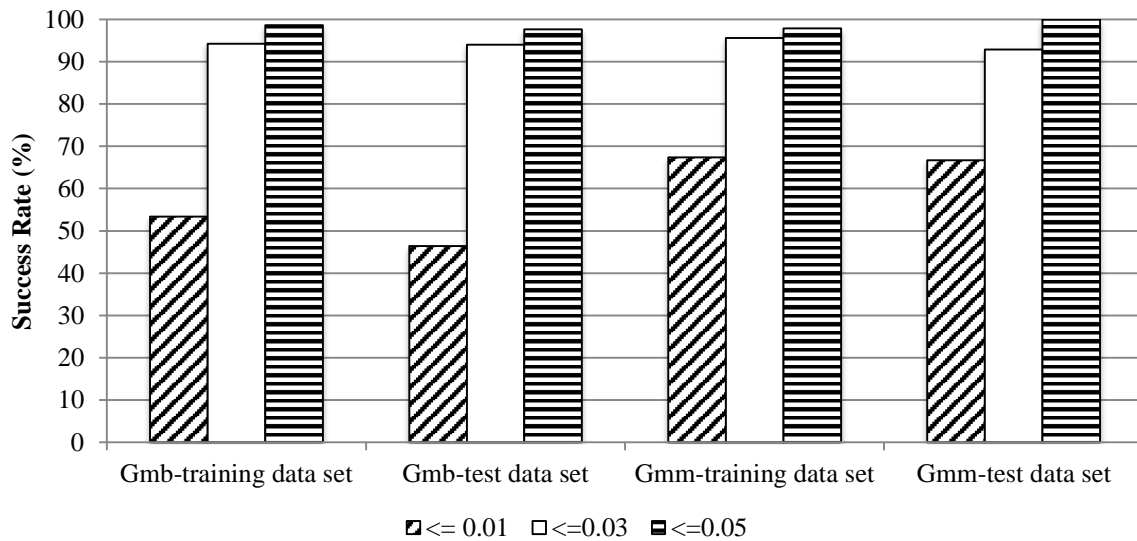


Figure 3: SR of the ANN model on the predictions of  $G_{mb}$  and  $G_{mm}$

Table 2: SR of the ANN model on the predictions of air void levels

	Error in % Va	SR % for Va
AV-training data set	< $\pm 0.5$	54.19
	< $\pm 1.0$	86.02
AV-test data set	< $\pm 0.5$	55.95
	< $\pm 1.0$	90.48

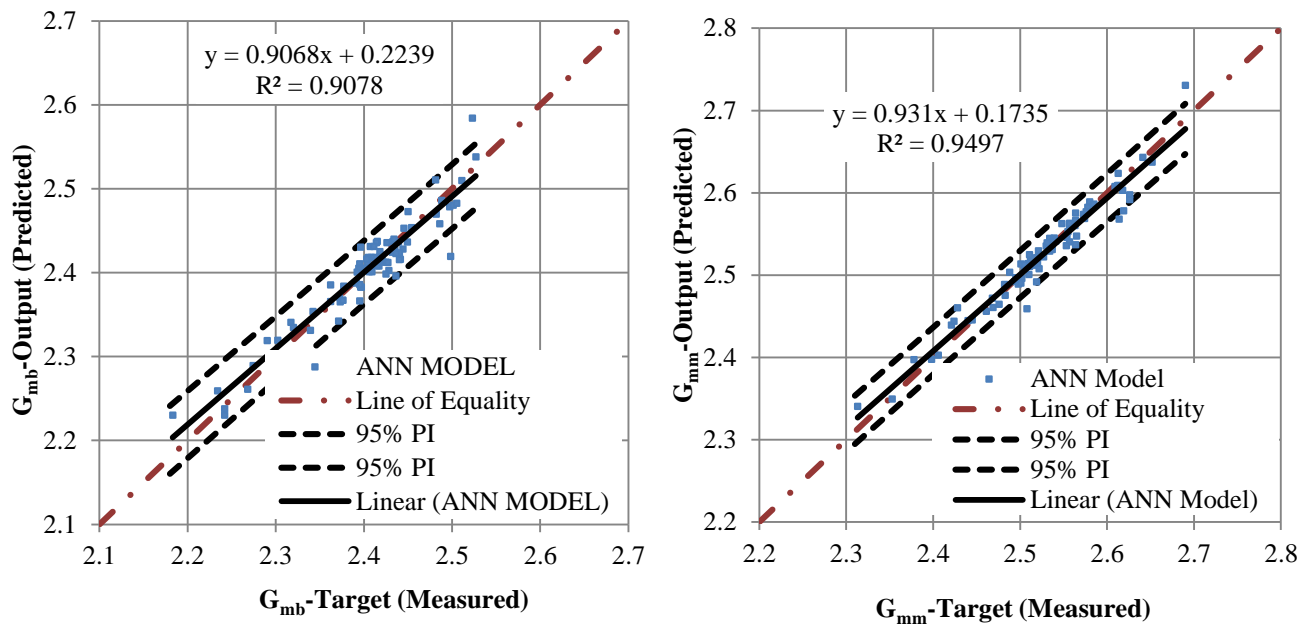


Figure 4: Predicted versus measured values of a)  $G_{mb}$ , b)  $G_{mm}$  for the testing data

The ANN model predictions for the test data reveals that the accuracy of the ANN model is significantly similar when the success of the training and testing datasets is compared. Additionally, the predictions are very reasonable since the large diversity of the mix data (i.e., different laboratories, different operators, wide range of mix gradations) is considered.

#### 4. POTENTIAL TIME SAVING

The major advantages of the ANN based Marshall Mix Design method is that it can potentially provide significant time, cost, and labor savings. Table 3 shows a timeline comparison of traditional and the ANN based Marshall mix design procedures. The traditional Marshall mix design takes approximately 5 days, whereas the Marshall mix design takes approximately either 1.5 days or 3 days if the ANN based model is used. In the proposed ANN based procedure, the users can vary the inputs (i.e., gradations, binder content) till the trial blend meets the agency requirements such as  $V_a$ , VMA and VFA. After determining the best blend that the model proposed, the mixture can be prepared and tested in the laboratory to determine the  $G_{mm}$  and  $G_{mb}$ . This is named as best case in Table 3 and will take approximately 1.5 days. There is approximately 55% probability that the  $V_a$  measurements will be within  $\pm 0.5$  error range, based on Table 2. In case the error between the measured and predicted  $V_a$  is larger than  $\pm 0.5\%$ , the laboratory experiments should be repeated at adjusted binder contents to validate the optimum binder content. The model has approximately 90% probability of predicting the error less than  $\pm 1\%$ . Thus, the overall procedure will take approximately 3 days. Since then, the time saving is between 2 to 3.5 days if ANN based model is used, as compared to the traditional mix design procedure.

#### 5. ANN TOOL FOR MARSHALL MIX DESIGNS

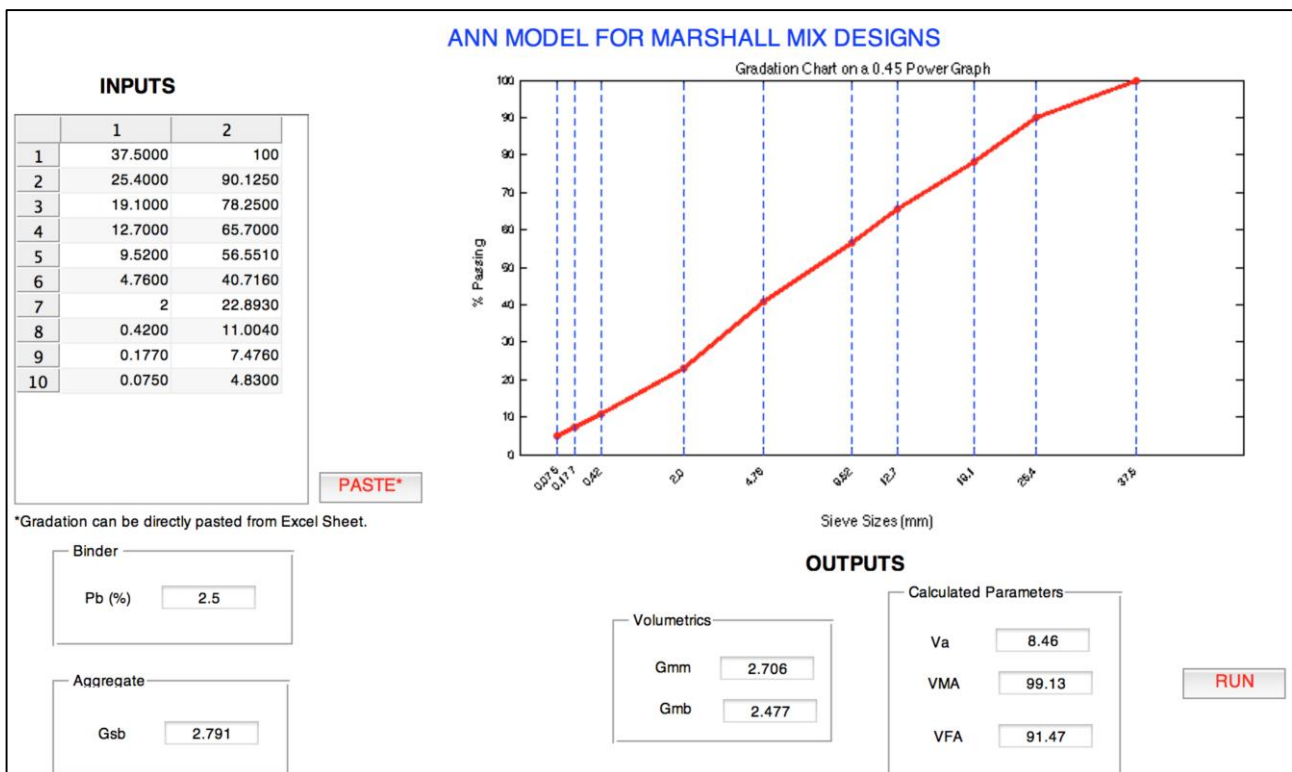
An interface was developed for the proposed ANN model as shown in Figure 5. ANN tool provides users to copy and paste the mix design gradations from Excel sheets, input the binder content, bulk specific gravity of aggregates. Once, the user press the 'RUN' button maximum bulk specific gravities of the mixture and theoretical maximum specific gravity are estimated by the model. In addition, volumetrics ( $V_a$ , VMA and VFA) are calculated and displayed on the screen in less than a few seconds.

As shown in Figure 6, two significantly different mix designs are randomly selected to compare the air voids versus binder contents curve. The first mix design has nominal maximum aggregate (NMAS) size of 25.4 mm, whereas the second mix has NMAS of 37.5 mm. For the first mix, the measured optimum binder content is determined to be 4.92% for 5.66% air void. Due to the predicted air voids with the help of ANN tool, the optimum binder content is estimated to be 5.03% for the exact amount of air voids (5.66%). Similarly, for the second mix design, the optimum binder content is determined to be 3.9% for 4.83% air void. Due to the predicted air voids with the help of ANN tool, the optimum binder content is estimated to be 3.92% for the exact amount of air voids (4.83%). As a result, the binder content differences between the predicted and measured are 0.11 and 0.02 for the first and second mix design, respectively. The air void versus binder content curves will answer the needs of the users.



**Table 3: Timeline of traditional and ANN based Marshall Mix Design Methods**

MARSHALL MIX DESIGN	
ITEM	DURATION (BUSINESS DAYS)
1 Aggregate Gradation + 5 different binder contents	~3.5 days
1 Aggregate Gradation + <i>Optimum</i> binder contents	+ ~1.5 days
	<b>~5.0 days</b>
ANN MODEL (Best Case)	
ANN Trials (Each trial takes about 1-2 seconds)	~0 days
1 Aggregate Gradation + <i>Optimum</i> binder contents (If Error $V_a \leq 0.5\%$ )	~1.5 days
	<b>~1.5 days</b>
ANN MODEL (Worst Case)	
ANN Trials (Each trial takes about 1-2 seconds)	~0 days
1 Aggregate Gradation + <i>Predicted</i> binder content (If Error $V_a > 0.5\%$ )	~1.5 days
1 Aggregate Gradation + <i>Adjusted</i> binder contents	~1.5 days
	<b>~3.0 days</b>



**Figure 5: Screenshot of the ANN tool**

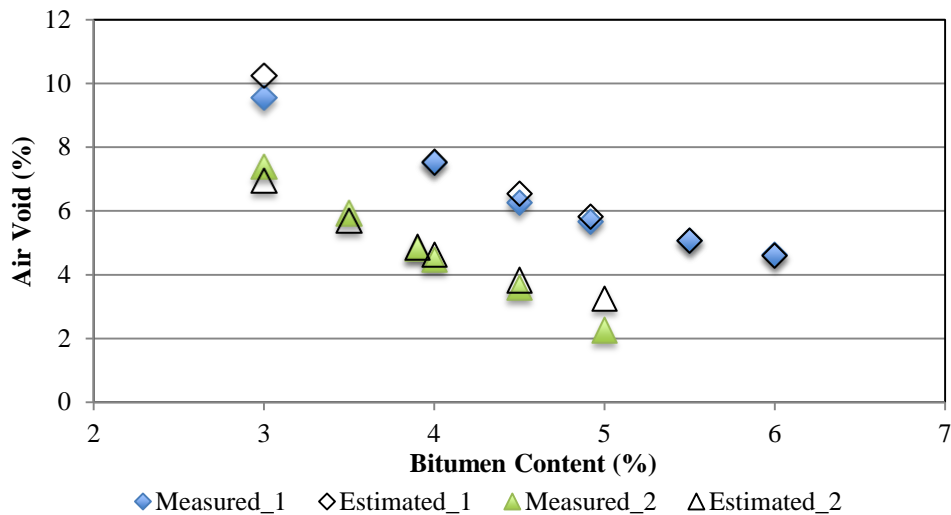


Figure 6: Screenshot of the ANN tool

## 6. CONCLUSION

A backpropagation ANN for asphalt mixture design was developed to predict the Marshall mixture volumetrics (i.e.  $V_a$ , VMA, VFA) by inputting aggregate gradation, binder content and bulk specific gravity of aggregates. The proposed ANN model is a computationally efficient model that can be used for estimating the optimum design properties of asphalt mixtures. It is proven that the traditional mix design period can be shortened between 2 to 3.5 days. This timeframe provides significant time, cost, and labor savings to designers. In addition, the agencies can use the ANN proposed model as a QC/QA tool. The tool will help the agencies to identify the problems in the mix designs, if there are errors and typos in the mix designs. Based on the analyses presented in this paper, the ANN model was able to predict specific gravities ( $G_{mb}$  and  $G_{mm}$ ) within acceptable accuracies. The air void prediction will be within  $\pm 0.5\%$  error limit at 55% of the time, which reduces the time of Marshall mix design from 5 to 1.5 days. If the error is higher than  $\pm 0.5\%$ , there is 90% probability that the error will be within  $\pm 1.0\%$  range. It will reduce the traditional Marshall mix design period from 5 days to 3 days. Although the results are promising, the model may be improved by including the number of blows to the inputs in the future studies.

## ACKNOWLEDGEMENTS

The authors would like to thank Turkish General Directorate of Highways for providing the Marshall mix design data.

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