Multivariate statistical analysis of low temperature properties obtained from laboratory tests on asphalt mixtures

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ABSTRACT

The paper presents multivariate statistical analysis on data obtained from low temperatures laboratory tests on Stone mastic asphalt (SMA) mixtures. Data were gathered from four renowned road laboratory institutes in Europe. Mainly two methods were used for statistical analysis of data: Principal component analysis (PCA) and Partial least squares (PLS).
The PCA method was used for studying interdependence between measured low temperature properties (TSRST, UTST and tensile strength reserve). With first two principal components 72.14% of variability in data was explained. With loading plot of first two principal components correlations between low temperature properties were presented. The scores plot was used to examine possible variation between different laboratories. We did not find any systematic difference between participating laboratories.
With PLS analysis dependence between input variables X (TSRST $\sigma cry,f$, TSRST Tf, UTST 5 °C, UTST −10 °C, UTST −25 °C) and output variables Y ($\Delta\beta tmax$, $T\Delta\beta tmax$) was evaluated. We found that UTST −10 °C and UTST −25 °C $\Delta\beta tmax$ are correlated with $\Delta\beta tmax$ and TSRST Tf is correlated with $T\Delta\beta tmax$.
The paper presents also a statistical model for prediction low temperature properties of asphalt. Model demands only three input variables (TSRST $\sigma cry,f$, TSRST Tf and UTST −10 °C) instead of five to get the same characterisation of asphalt low temperature properties.

Keywords: Asphalt, Low-Temperature, Testing

1. INTRODUCTION

Statistics is the study of collection, analysis, interpretation, presentation, and organization of data [1]. The asphalt laboratories for quality control perform several tests on produced asphalt mixtures. Test results are evaluated with statistic tools. Statistics is used in asphalt industry as a mean for identifying quality anomalies at asphalt mixture production. In our study, statistical methods were used to determine main factors affecting on low temperature properties of asphalt. Standardized test for determination resistance of asphalt pavement to low temperatures is using physical fact that at low temperatures asphalt pavement is contracting. If the contraction due to cooling is prevented, than with falling temperatures tensile stress is increasing. When maximum tensile strength in the asphalt specimen is reached, fracture (micro-cracking in the binder matrix) occurs [2]. On the asphalt pavement such damages appear primarily due to temperature changes, changes in bituminous binder, excessive traffic load and/or deficiencies in construction and maintenance. So, the knowledge about the behaviour of cracks is essential both for researchers, planners and civil engineers [3, 4, 5].

We are aware that low temperature cracking occurs largely due to loss of durability during cold storage and during time in service and so neither of these are captured by the TSRST tests or the UTST. On the other hand these tests can simulate crack initiation as important part of crack propagation process. But these tests certainly does not have any relationship with physical and chemical hardening of bitumen, that have an overbearing influence on long term resistance of asphalt on low temperatures.

This paper presents multivariate statistical analysis based on results obtained from two Tensile Stress Restrained Specimen Test (TSRST) and Uniaxial Tensile Strength Test (UTST) in accordance with standard EN 12697-46 on Stone mastic asphalt (SMA) mixtures. Principal component analysis (PCA) and Partial least squares (PLS) were performed by software XLSTAT (Microsoft Excel). With statistical method PCA we analysed interdependence between results of tests at low temperature (TSRST, UTST and tensile strength reserve). PLS analysis was used to evaluate dependence between input variables X (TSRST $\sigma_{cry,f}$, TSRST T_f, UTST 5 °C, UTST −10 °C, UTST −25 °C) and output variables Y ($\Delta\beta_{tmax}$, $T_{\Delta\beta tmax}$). At the end of statistical model for prediction low temperature properties of asphalt is presented.

2. DATA

The data for multivariate statistical analysis of SMA is obtained from literature [6] and from some laboratories performing low temperatures tests: TU Braunschweig (D), TU Wien (A), Ramtech (CRO) and ZAG (SI). Table 1 shows the results of TSRST and UTST tests. According to the standard EN 12697-46 [7] UTST test is carried out at temperature T = 20, 5, -10 and -25 °C, but due to the fact that some of the laboratories don't carry out the UTST at temperature T = 20°C, this temperature was not included in this analysis. From TSRST we obtained tension stress at failure $\sigma_{cry,f}$ and temperature at failure T_f, from both tests we calculated tensile strength reserve $\Delta\beta t(T)$ [6] and corresponding temperature $T_{\Delta\beta tmax}$. Equation for tensile strength reserve is:

$$\Delta\beta_t(T) = \beta_t\,(T) - \sigma_{cry}(T) \qquad (1)$$

In Table 1 we can see that all test results are in the range $x \pm 3 \cdot s$. First Grubbs test [8] was performed to detect outliers. In column RESERVE $\Delta\beta_{tmax}$, there is at least one minimum extreme value that deviates from normally distributed population. Since the value of G_{min} in column RESERVE $\Delta\beta_{tmax}$ doesn't deviate much from the required G_{min} (< 2.9033), this extreme is not removed from the population.

Table 1: Results of test at low temperatures for SMA

Seq. no.	Lab.	Asphalt mixture with type of bitumen (or brand of bitumen)	TSRST $\sigma_{cry,f}$ [MPa]	TSRST T_f [°C]	UTST at 5 °C [MPa]	UTST at −10 °C [MPa]	UTST at −25 °C [MPa]	RESERVE $\Delta\beta_{tmax}$ [MPa]	RESERVE $T_{\Delta\beta tmax}$ [°C]
1	TU BRAUNSCHWEIG	SMA 11s (Vilabit 65)	3.94	-32.9	1.41	4.87	4.13	4.66	-11.5
2		SMA 11s (Bitupol c)	4.65	-33.3	1.99	6.03	5.45	5.62	-11.5
3		SMA 11s (Zalaplast II)	4.57	-32.3	1.84	5.4	4.83	5.01	-11.2
4		SMA 11s (Bitupol c)	4.51	-31.3	2.5	5.56	5.14	4.95	-10.6
5		SMA 8s (Bitupol b)	4.79	-30.6	2.49	6.62	5.07	6.07	-10.3
6		SMA 11s (Polyplast A1)	4.28	-28.2	2.59	6.33	5.26	5.51	-10
7		SMA 8s (Vilabit 65)	4.67	-32.8	1.85	5.04	5.46	4.61	-12.5
8		SMA 8s (Olexobit 45)	3,68	-23.9	3.19	4.68	3.53	3.77	-4.9
9		SMA 8s (Olexobit 45)	2,57	-24	1.96	2.93	2.83	2.09	-5.2

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10		SMA 8s (Olexobit 45)	2,77	-25.1	2.42	3.9	3.37	3.13	-6.9
11		SMA 8s (Olexobit 45)	4,29	-26	3.27	5.34	3.73	4.44	-6.5
12		SMA 11	3,585	-31.4	1.373	3.865	4.423	3.47	-12.9
13		SMA 11 PmB 45/80-65	4,959	-31	2.295	6.56	5.125	5.839	-10.3
14		SMA 11 S I Pmb 45A	4,053	-25.5	2.795	4.569	3.983	3.697	-6.5
15		SMA 11 S II Pmb 45A	3,848	-25.567	2.757	3.87	3.715	3.103	-3.7
16	TU WIEN	SMA 11s (Vilabit 65)	4,09	-34.9	1.77	4.14	4.91	3.89	-15.1
17		SMA 11s (Bitupol c)	5,11	-33.5	2.85	5.78	5.64	5.07	-11.1
18		SMA 11s (Zalaplast II)	4,01	-34.1	2.01	4.75	4.25	4.48	-11.4
19		SMA 11s (Bitupol c)	4,25	-31.7	2.48	4.61	4.69	4.02	-10.3
20		SMA 8s (Bitupol b)	4,28	-32.1	2.08	4.41	4.36	3.81	-10.2
21		SMA 11s (Polyplast A1)	3.58	-29.5	2.06	4.99	5.16	4.19	-11.2
22		SMA 11 B70/100	3.5	-30.5	2.54	4.47	4.43	3.87	-13
23	RAMTECH	SMA 8 PmB 45/80-65	4.61	-27.77	2.44	6.21	4.37	5.28	-11.2
24		SMA 11s -004	4.39	-26.53	2.14	5.8	3.99	5.08	-11.1
25		SMA 11s-PmB III	5.25	-24.53	2.91	4.88	3.95	3.72	-6.2
26		SMA 11s	3.46	-30.27	2.31	5.86	3.53	5.51	-11.8
27		SMA 11 PmB 45/80-65	3.9	-32.57	2.07	6.31	5.29	6.12	-13.9
28		SMA 11 PmB 45/80-65	4.41	-29.8	2.12	5.84	4.36	5.15	-11.9
29		SMA 11 PmB 45/80-65	4.43	-30.37	2.12	6.09	5.03	5.72	-13.1
30		SMA 11 PmB 45/80-65	4.87	-28.1	2.66	6.06	5.19	5.39	-11.3
31		SMA 11 PmB 45/80-65	3.26	-33.4	2.37	5.17	4.39	5.13	-13.7
32		SMA 11 PmB 45/80-65	6.15	-29.6	2.83	6.79	5.22	5.38	-10.9
33		SMA 8 PmB 45/80-65	6.03	-25.83	1.97	6.68	4.3	5.58	-11.1
34		SMA 11 PmB 45/80-65	3.31	-29	2.19	5.7	3.21	5.33	-11.6
35		SMA 11 PmB 45/80-65	4.6	-26.8	1.92	5.82	4.42	5.24	-11.8
36		SMA 11 PmB 45/80-65	4.86	-30.7	2.63	6.77	5.65	6.06	-12.6
37		SMA 8 PmB 45/80-65	3.35	-31.43	2.14	5.8	3.99	4.66	-10.1
38		SMA 8 PmB 45/80-65	2.68	-24.93	2.28	5.13	4.37	4.17	-12.3
39	ZAG	SMA 8 PmB 45/80-65	5.018	-33.4	2.045	6.341	5.87	6.001	-13.9
40		SMA 11 PmB 45/80-65	5.131	-29.7	2.978	6.504	6.365	5.654	-12.4
41		SMA 8 PmB 45/80-65	4.45	-32.8	1.917	5.468	4.911	4.859	-13.2
42		SMA 11 PmB 45/80-65	2.852	-29.8	3.759	6.578	5.961	5.121	-13.9
43		SMA 11 PmB 45/80-65	3.022	-29.9	3.554	5.9	5.457	4.392	-12.2
44		SMA 11 PmB 45/80-65	2.862	-31.1	3.393	6.398	6.398	5.149	-15.6
STATISTICS									
Number of measurements – n			44	44	44	44	44	44	44
Average value – x			4.156	-29.739	2.392	5.473	4.675	4.773	-10.968
Standard deviation – s			0.840	3.044	0.529	0.932	0.834	0.916	2.681
Maximum value – x_{max}			6.15	-23.9	3.759	6.79	6.398	6.12	-3.7
Minimum value – x_{min}			2.57	-34.9	1.373	2.93	2.83	2.09	-15.6
Range – R			3.580	11.000	2.386	3.860	3.568	4.030	11.900
$x + 3 \cdot s$			6.678	-20.607	3.980	8.268	7.176	7.522	-2.924
$x - 3 \cdot s$			1.635	-38.870	0.805	2.678	2.175	2.023	-19.012
G_{min} (< 2.9033; α = 0.05)			1.888	1.696	1.926	2.730	2.214	2.927	1.727
G_{max} (< 2.9033; α = 0.05)			2.372	1.918	2.582	1.414	2.067	1.470	2.711

3. PRINCIPAL COMPONENT ANALYSIS (PCA)

3.1. General

Principal component analysis (PCA) [9] is an orthogonal linear transformation of data to a new coordinate system. The greatest variance by some projection of the data lies on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on.

Fig. 1 is an example of the two-dimensional space of original variables X_1 and X_2, together with its main corresponding principal components Y_1 and Y_2. Since the correlation between X_1 and X_2 is large, Y_1 can successfully replace both starting variables X_1 in X_2. Two-dimensional space is transformed into a one-dimensional, with minimum loss of information.

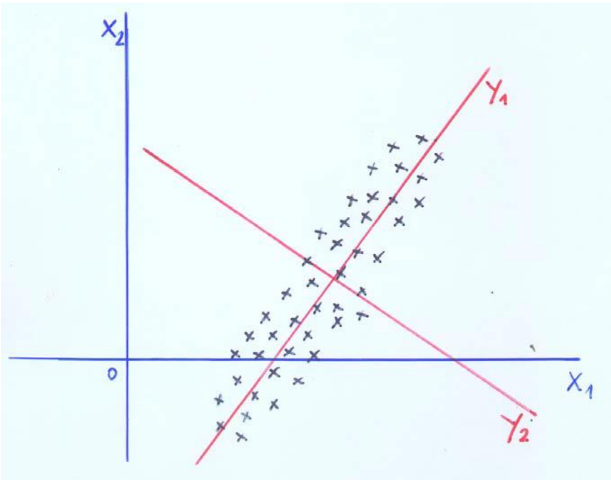


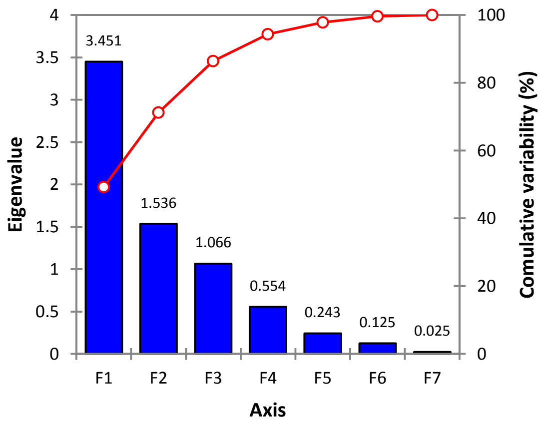
Figure 1: X_1 and X_2 are the original variables; the data is represented by points. Y_1 and Y_2 are the corresponding principal components. Two-dimensional space can be reduced to the one-dimensional space defined by Y_1 [10].

3.2. Analysis and results

Principal component analysis (PCA) is carried out to establish the interdependence of variables TSRST $\sigma_{cry,f}$, TSRST T_f, UTST 5 °C, UTST −10 °C, UTST −25 °C, $\Delta\beta_{tmax}$ and $T_{\Delta\beta tmax}$. Table 2 presents the results of the analysis: eigenvalues and cumulative variability in relation to the principal components. Variability of first two principal components (*F1* and *F2*) is 71.24 %, which is satisfactory. On the Fig. 2a are presented eigenvalues and cumulative variability and on Fig. 2b is loading plot of variables in the space of first two principal components. On Fig. 2b the first group forms *TSRST* T_f and $T_{\Delta\beta tmax}$, second *TSRST* $\sigma_{cry,f}$, *UTST −10 °C, UTST −25 °C* and $\Delta\beta_{tmax}$, and between them in third group is *UTST 5 °C.*

Table 2: Eigenvalues

Factors	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*F7*
Eigenvalues	3,451	1,536	1,066	0,554	0,243	0,125	0,025
Variability [%]	49,298	21,941	15,230	7,912	3,475	1,787	0,357
Cumulative [%]	49,298	71,239	86,469	94,381	97,856	99,643	100,000



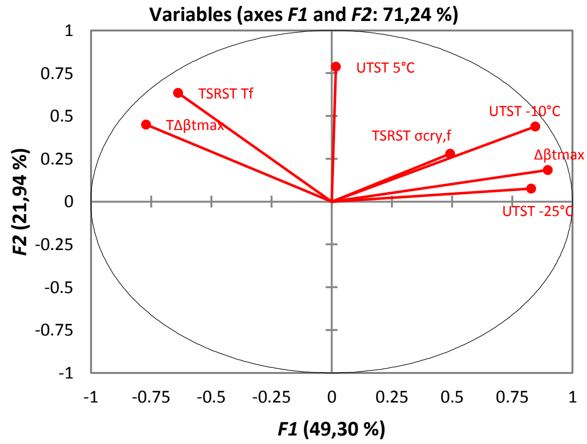


Figure 2a: Diagram of eigenvalues and cumulative variability

Figure 2b: Loading plot of variables in the space of first two principal components

After PCA, obtained values of factors *F* have been split by the laboratories. On Fig 3 we can find score plot of observations in the space of first two principal components. The results of each laboratory are presented with ellipsoid clusters, which have a center of gravity in the middle of the observation. It can be seen that the clusters overlap and from this we can assume that none of laboratories have significant systemic error at performing low temperature test. Due to the fact that overlap between the clusters is the smallest, it can be concluded that moderate difference was found between results from ZAG and from TU Wien. With additional study we found some reasons for that difference [5]. The largest cluster on Fig. 3 belongs to TU Braunschweig, which shows the highest variability between the measurements. The reason for such high variability can be higher variability of tested materials (e.g. the type of bitumen, a mixture of stone aggregate) or lower accuracy of measurements.

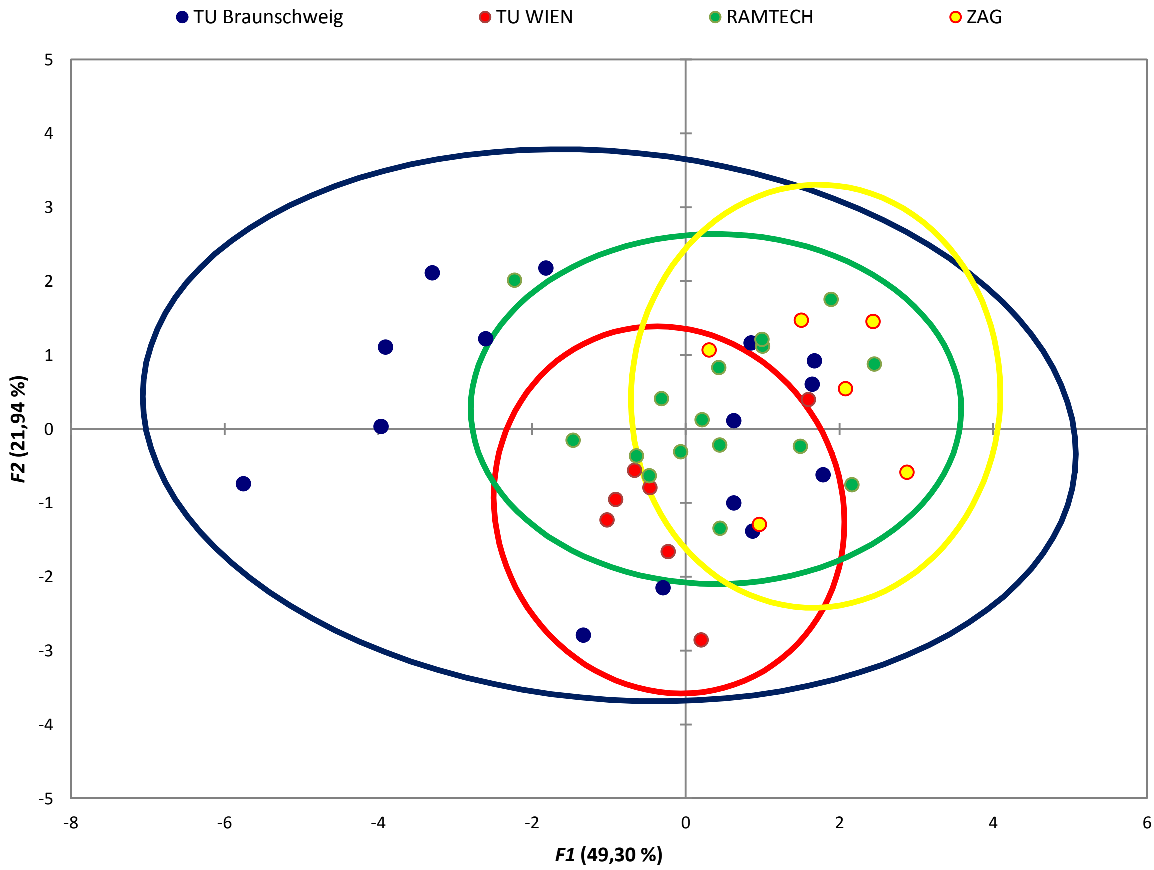


Figure 3: Score plot of observations in the space of first two principal components

4. PARTIAL LEAST SQUARES (PLS)

4.1. General

Partial least squares regression (PLS regression) [10] is a statistical method that is used to find a linear regression model by projecting the predicted variables and the observable variables to a new space. PLS regression is useful when the matrix of predictors has more variables than observations.

Figure 4 shows a geometric representation of PLS regression [11]. Mean-centered and scaled fictitious X and Y data sets are illustrated as a cloud of points in each variable space. Only three variable axes are displayed respectively (X_1, X_2, X_3 and Y_1, Y_2, Y_3). Use of PLS regression analysis results in linear combinations of the original X and Y variables, respectively, which creates new or 'latent' variables (T_1 and U_1). These latent variables - also called X and Y scores - are essentially identical to principal components. The PLS regression model improves the relationship between the X and Y axes because the iterative algorithm used, exchanges scores between the two data sets and therefore, defines the latent variables in the X data set that have high covariance with those in the Y data set. Covariance is sought in each dimension, and once it is found in one dimension, the X data set is decomposed at the same time as the predicted Y data set is created. In essence, PLS regression simultaneously projects the X and Y variables onto a common subspace (TU) in such a manner that there is a close relationship between the position of one observation on the X plane and its corresponding position on the Y plane. This approach creates a PLS regression component for the first modeled dimension T_1 and U_1 [11].

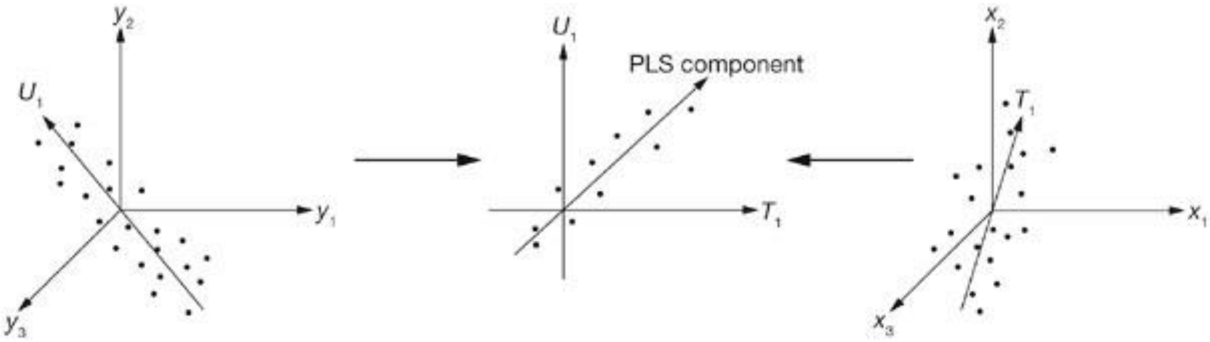


Figure 4: A geometric representation of partial least squares (PLS) regression [11]

4.2. Analysis and results

With multivariate statistical method of Partial least squares (PLS) we are checking the impact of the input variables X (*TSRST* $\sigma_{cry,f}$, *TSRST* T_f, *UTST 5 °C, UTST −10 °C, UTST −25 °C*) and output variables Y ($\Delta\beta_{tmax}$, $T_{\Delta\beta tmax}$). Software XLSTAT was used to perform PLS. Table 3 and Figure 5a are presenting quality of PLS models. With first two components we found Q^2 = 67.4 % and the average R^2_Y and R^2_X is R^2 = 69.7%, which is satisfactory. Table 4 tabular presents matrix of correlations between the variables with the components t in relation to the variables X and Y. Correlations of the variables with first two components are graphically presented in Figure 5b. It can be seen that *UTST -10°C* and *UTST -25°C* are near to $\Delta\beta_{tmax}$ and *TSRST* T_f is the close to $T_{\Delta\beta tmax}$, which corresponds to the PCA results.

Table 3: Model quality

Index	Comp1	Comp2	Comp3	Comp4	Comp5
Q^2 cum	0,565	0,674	0,788	0,792	0,791
R^2_Y cum	0,599	0,729	0,841	0,849	0,855
R^2_X cum	0,431	0,665	0,738	0,953	1,000

Table 4: Correlation matrix of the variables with the *t* components

Variables	*t1*	*t2*	*t3*	*t4*	*t5*
TSRST $\sigma_{cry,f}$	0,555	0,528	−0,213	0,602	−0,072
TSRST T_f	−0,657	0,668	0,132	−0,182	−0,266
UTST 5 °C	0,042	0,526	−0,278	−0,733	0,327

UTST −10 °C	0,812	0,402	0,384	−0,168	0,056
UTST −25 °C	0,868	−0,067	−0,280	−0,337	−0,224
$\Delta\beta_{tmax}$	0,848	0,192	0,428	0,026	0,073
$T_{\Delta\beta tmax}$	−0,692	0,471	−0,201	0,129	0,081

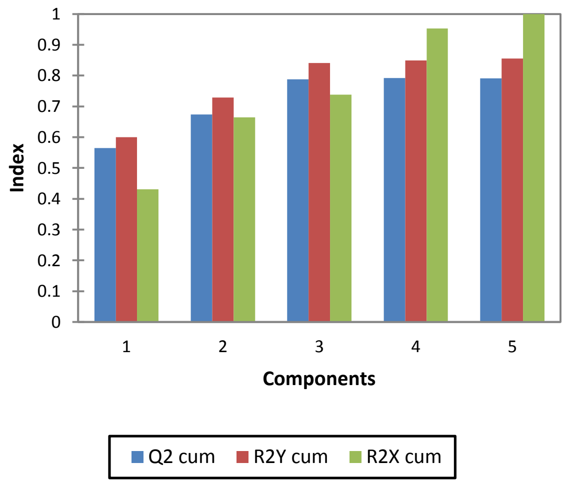


Figure 5a: Model quality by number of components

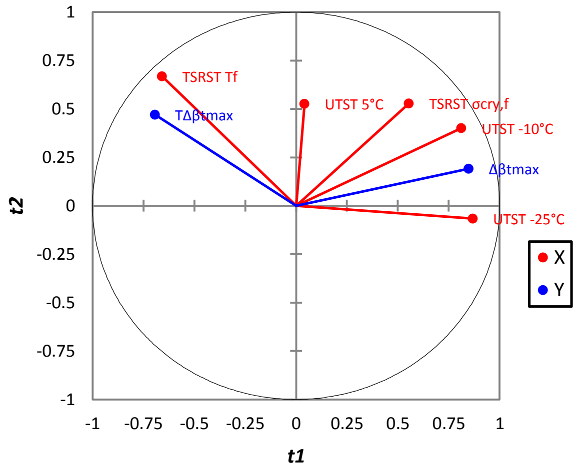
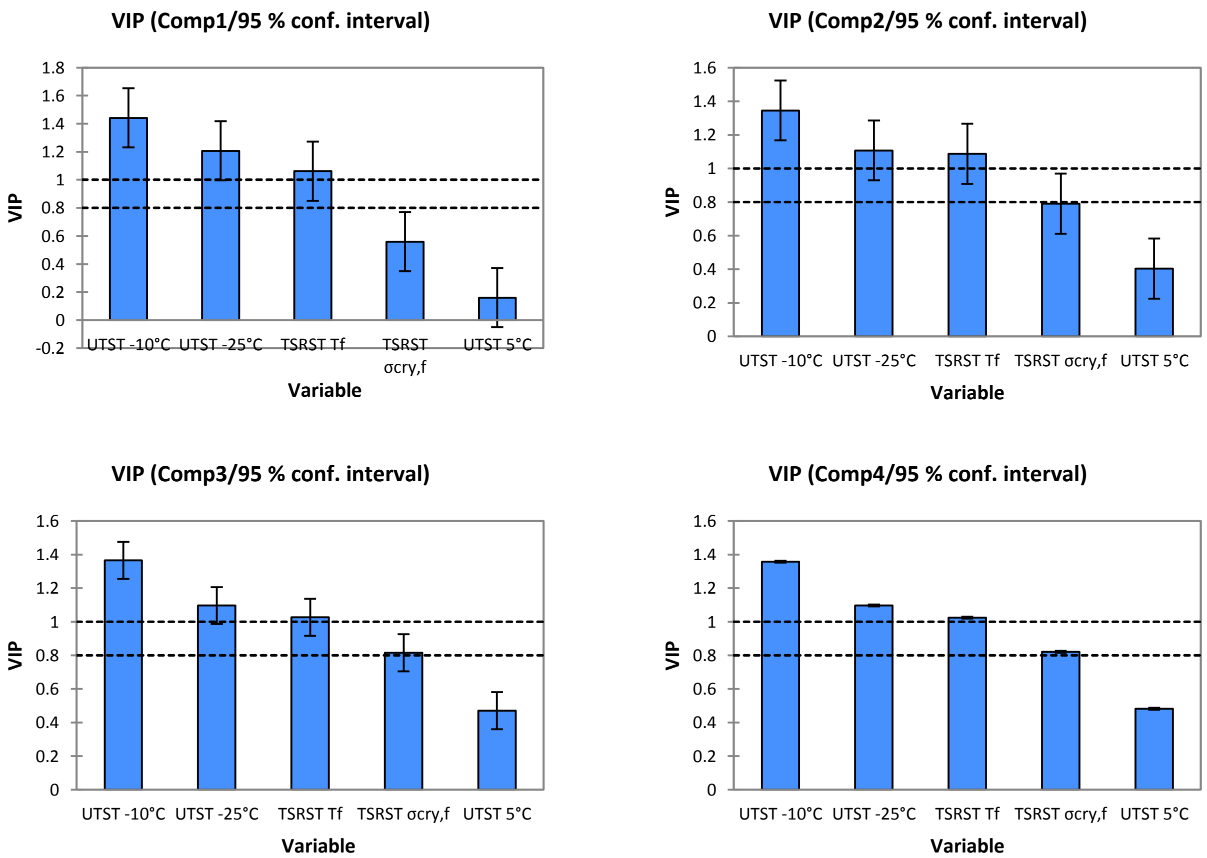


Figure 5b: Correlations of the variables with first two components (*t1* and *t)*

Fig. 6 presents the VIPs (Variable Importance for the Projection) at the 95 % confidence for each input (explanatory) variable, for an increasing number of components. This allows quick identification of explanatory variables that contribute most to the models. On the graphs (Fig. 6) we can see that components *UTST −10 °C*, *UTST −25 °C* and *TSRST* T_f (above 1.0 VIP) are the most important variables in the projections and less important components are *UTST 5°C* and *TSRST* $\sigma_{cry,f}$ (under 1.0 VIP), which are good candidates to be excluded from the model.



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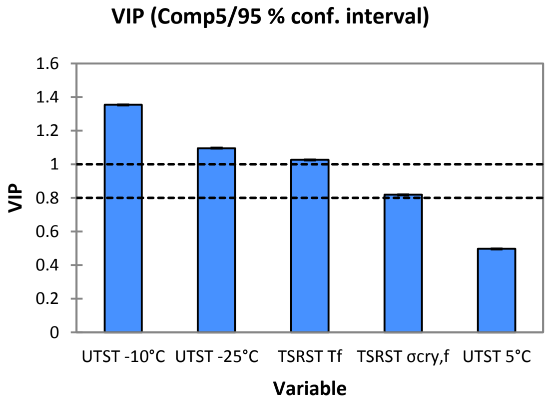

Figure 6: Variable importance in the projection (VIP)

4.3. Statistical model

In this part a statistical model obtained with ordinary multiple linear regression for prediction low temperature properties of asphalt is presented. The final result is equation of statistical model for the output (dependent) variable $\Delta\beta_{tmax}$ and $T_{\Delta\beta tmax}$ as a function of input (explanatory) variables. Equations of the model for maximum tensile strength reserve $\Delta\beta_{tmax}$ and temperature of maximum tensile strength reserve $T_{\Delta\beta tmax}$ have the next form:

$$\Delta\beta_{tmax} = -1{,}15 + 6{,}79E-03 \cdot (TSRST\ \sigma_{cry,f}) - 5{,}81E-02 \cdot (TSRST\ T_f) - 0{,}29 \cdot (UTST\ 5\,°C) + 0{,}98 \cdot (UTST\ -10\,°C) - 0{,}10 \cdot (UTST\ -25\,°C) \quad (R^2 = 0{,}945) \tag{2}$$

and

$$T_{\Delta\beta tmax} = 0{,}64 + 1{,}29 \cdot (TSRST\ \sigma_{cry,f}) + 0{,}28 \cdot TSRST\ T_f + 1{,}91 \cdot (UTST\ 5\,°C) - 1{,}28 \cdot (UTST\ -10\,°C) - 1{,}36 \cdot (UTST\ -25\,°C) \quad (R^2 = 0{,}765) \tag{3}$$

The analysis shows a very good correlation between the model ($R^2 = 0.945$) for the output variable $\Delta\beta_{tmax}$ and slightly lower ($R^2 = 0.765$) for $T_{\Delta\beta tmax}$, but still satisfactory.

We have also taken into account VIP determined with PLS and we prepared a new rational statistic model for prediction low temperature properties of asphalt. For a new statistic model only three input variables (TSRST $\sigma_{cry,f}$, TSRST T_f and UTST −10 °C) are taken into account instead of five to get the same characterisation of asphalt low temperature properties. From linear model we found that output variable $\Delta\beta_{tmax}$ still has a very good correlation ($R^2 = 0.914$) although variable $T_{\Delta\beta tmax}$ has slightly worse correlation ($R^2 = 0.665$) as original model, but still satisfactory. With this new rational model for the low temperature tests we found that price of the total costs can drop for approximately 50 %. Equations for the new model with three variables are the following ones:

$$\Delta\beta_{tmax} = -2{,}06 + 5{,}77E-02 \cdot (TSRST\ \sigma_{cry,f}) - 0{,}07 \cdot (TSRST\ T_f) + 0{,}85 \cdot (UTST\ -10\,°C) \quad (R^2 = 0{,}914) \tag{4}$$

$$T_{\Delta\beta tmax} = 9{,}51 + 0{,}93 \cdot (TSRST\ \sigma_{cry,f}) + 0{,}56 \cdot (TSRST\ T_f) - 1{,}38 \cdot (UTST\ -10\,°C) \quad (R^2 = 0{,}665) \tag{5}$$

5. CONCLUSION

Multivariate statistical analysis of data obtained from low temperatures laboratory tests on Stone mastic asphalt (SMA) mixtures was performed. The PCA method was used for studying interdependence between measured low temperature properties (TSRST, UTST and tensile strength reserve). With first two principal components 72.14% of variability in data was explained. With loading plot of first two principal components, correlations between low temperature properties were presented. The scores plot was used to examine possible variations between different laboratories. We did not find any systematic difference between participating laboratories.

With PLS analysis dependence between input variables X (TSRST σcry,f, TSRST Tf, UTST 5 °C, UTST −10 °C, UTST −25 °C) and output variables Y (Δβtmax, TΔβtmax) were evaluated. We found that UTST −10 °C and UTST −25 °C Δβtmax are correlated with Δβtmax and TSRST Tf is correlated with TΔβtmax.

Finally statistical model for prediction low temperature properties of asphalt was calculated. Model demands only three input variables (TSRST σcry,f, TSRST Tf and UTST −10 °C) instead of five to get the same characterisation of asphalt low temperature properties.

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